Exercise 20

Evaluate the integral.

$$\int_{-1}^{1} x^{100}\,dx$$

Solution

According to part 2 of the fundamental theorem of calculus,

$$\int_a^b f(x)\,dx = F(b) - F(a),$$

where F is an antiderivative of f. Use the power rule in reverse here: Bump up the exponent by 1 and divide by that exponent.

$$\begin{aligned}\int_{-1}^{1} x^{100}\,dx &= \frac{x^{101}}{101}\bigg|_{-1}^{1} \\ &= \frac{1^{101}}{101} - \frac{(-1)^{101}}{101} \\ &= \frac{1}{101} - \frac{-1}{101} \\ &= \frac{2}{101}\end{aligned}$$